# Introduction to Digital Signal Processing

## Signals

Continuous time (CT) signal: .

Discrete time (DT) signal: .

Causal signals are signals (CT) which only have value when , or

|  |  |  |
| --- | --- | --- |
|  |  |  |

Causal signals for DT are similar to CT.

## Energy and Power

|  |  |  |
| --- | --- | --- |
|  | **Continuous time** | **Discrete time** |
| **Periodic** |  |  |
| **Aperiodic** |  |  |

Note that:

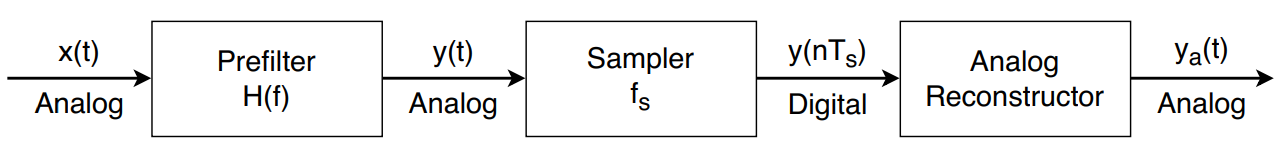
1. A non-periodic signal maybe energy signal. If is finite, then the signal is called as energy signal.
2. A periodic signal maybe power signal. If is finite, then the signal is called as power signal.
3. If a signal is summation of sine signal with amplitude and cosine signal with amplitude , then the power of this signal is given by

|  |  |  |
| --- | --- | --- |
|  |  |  |

# Sampling and Reconstruction

## Overview

Given the input signal if form of summation of its frequency components . Then the sampling and reconstruction procress follows the below figure



## Prefiter Process

Normally, there are 3 types of filter which are used in prefilter process

1. No prefilter:
2. Ideal low pass filter with cut off frequency :

|  |  |  |
| --- | --- | --- |
|  |  |  |

1. Practical low pass filter with cut off frequency (To easier we make the assumption ):

|  |  |  |
| --- | --- | --- |
|  |  |  |

Where is the attenuation of signal at -th frequency component. In more detail:

* is the number of octave from to .
* is the attenuation of the filter after cut off frequency.
* is the number of decade from to .
* is the attenuation of the filter after cut off frequency.

## Sampling Process

The process of sampling the CT signal at rate is the process of taking value of original signal each period time of or

|  |  |  |
| --- | --- | --- |
|  |  |  |

To fully reconstruct the signal must be band limited signal, the sampling rate should choose follow the Nyquist theorem, that is, .

## Reconstruction Process

The Nyquist interval (NI):

|  |  |  |
| --- | --- | --- |
|  |  |  |

If the -th frequency component of the signal **belongs to NI** then the analog reconstructed frequency is . If the-th frequency component of the signal is **beyond the NI** then the analog reconstruct ted frequency is .

# Quantization Process

## Parameters

Analog signal range

|  |  |  |
| --- | --- | --- |
|  |  |  |

Quantization bit

|  |  |  |
| --- | --- | --- |
|  |  |  |

Quantization resolution

|  |  |  |
| --- | --- | --- |
|  |  |  |

Mean error (Expectation error) ()

|  |  |  |
| --- | --- | --- |
|  |  |  |

Second moment error ()

|  |  |  |
| --- | --- | --- |
|  |  |  |

RMS error

|  |  |  |
| --- | --- | --- |
|  |  |  |

Noise variance or average noise power

|  |  |  |
| --- | --- | --- |
|  |  |  |

Normalized Signal to Noise Ratio

|  |  |  |
| --- | --- | --- |
|  |  |  |

Non Normalized Signal to Quantization Noise Ratio

|  |  |  |
| --- | --- | --- |
|  |  |  |

## Over-Sampling and Noise Shaping

### Over-Sampling without Using Noise Shaping

Over-sampling ratio

|  |  |  |
| --- | --- | --- |
|  |  |  |

Bit reduce

|  |  |  |
| --- | --- | --- |
|  |  |  |

### Over-Sampling with -th order Noise Shaping Filter

Bit reduce

|  |  |  |
| --- | --- | --- |
|  |  |  |

## DAC/ADC

Given a sequence with bits input, that is, the DAC will convert this sequence to quantized signal .

### Conversion Types

|  |  |
| --- | --- |
| **Type** | **Relationship** |
| Natural Binary |  |
| Offset Binary |  |
| 2’s Complement |  |

### Conversion Code Table

Let be the decimal value corresponding with a binary value and . Then the conversion code table is built as follows

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Natural | | Offset | | 2’s C |
|  |  |  |  |  |
|  |  |  |  |  |  |
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# Analysis of LTI Systems

## System Classification by Energy

|  |  |
| --- | --- |
| **Type** | **Relationship** |
| Passive system |  |
| Lossless system |  |
| Active system |  |

## Properties of LTI System

### Causality

A causal system is a system that output only depends on present and past value of input .

### Linearity

A system is called linear system if and only if it satisfies the condition

|  |  |  |
| --- | --- | --- |
|  |  |  |

Check for linearity:

* Step 1:

|  |  |  |
| --- | --- | --- |
|  |  |  |

* Step 2:

|  |  |  |
| --- | --- | --- |
|  |  |  |

* Step 3:

Compare and , if it equals, conclude that the system is linear.

### Time Invariant

A time-varying system is one whose parameters vary with time.

Check for time invariant:

* Step 1:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Calculate: (delay the ouput).

* Step 2:

|  |  |  |
| --- | --- | --- |
|  |  |  |

* Step 3:

Compare and , if it equals, conclude that the system is time invariant.

The checking process is similar for CT system

Bounded-input Bounded-output (BIBO) Stable

### BIBO Stable System (Stability)

If the system has bounded for all input is finite) which leads to all output is bounded is finite) then the system is said to be BIBO system.

Check for BIBO system: Assume that is finite. Calculate , if we can prove that is finite, we can conclude that the system is BIBO system.

**Discrete time BIBO System:** If the impulse response of the discrete time system is absolutely integrable, the system is said to be BIBO stable.

|  |  |  |
| --- | --- | --- |
|  |  |  |

## I/O Relationship

### Impulse Response

When , the output or the response of the system is called impulse response or .

### Difference Equation

|  |  |  |
| --- | --- | --- |
|  |  |  |

### Block Diagram

